International Journal of Mechanical and Industrial Technology **ISSN 2348-7593 (Online)**
Vol. 10, Issue 2, pp: (26-29), Month: October 2022 - March 2023, Available at: www.researchpublish.com

Methods for Solving a Problem Involving Fractional Algebraic Equations

Chii-Huei Yu

School of Mathematics and Statistics, Zhaoqing University, Guangdong, China

DOI: https://doi.org/10.5281/zenodo.7625086

Published Date: 09-February-2023

Abstract: **In this paper, based on a new multiplication of fractional analytic functions, we solve a problem involving fractional algebraic equations. The solutions of this problem can be obtained by using some methods. In fact, our results are generalizations of the results of ordinary algebraic equations.**

Keywords: **New multiplication, Fractional analytic functions, Fractional algebraic equations.**

I. INTRODUCTION

With the development of science and technology, especially since the 20th century, the theory and application of fractional calculus began to be widely concerned. Fractional calculus has been used in the research of physics, mechanics, electrical engineering, viscoelasticity, control theory, biology, economics, and other fields [1-11].

In this paper, based on a new multiplication of fractional analytic functions, we study a problem involving fractional algebraic equations. Using some techniques, we can solve this problem. On the other hand, our results are generalizations of the results of traditional algebraic equations.

II. PRELIMINARIES

Definition 2.1 ([12])**:** Let x and a_k be real numbers for all k, and $0<\alpha\leq 1$. If the function $f_\alpha:[a,b]\rightarrow R$ can be expressed as an α-fractional power series, i.e., $f_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}x^{k\alpha}$, then we say that $f_\alpha(x^\alpha)$ is α-fractional analytic function.

Next, a new multiplication of fractional analytic functions is introduced.

Definition 2.2 ([13])**:** If $0<\alpha\leq 1$. Suppose that $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ are two α-fractional analytic functions,

$$f_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}x^{k\alpha}=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes k}, \tag{1}$$

$$g_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}x^{k\alpha}=\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes k}. \tag{2}$$

Then

$$\begin{aligned}&f_\alpha(x^\alpha)\otimes g_\alpha(x^\alpha)\\&=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}x^{k\alpha}\otimes\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}x^{k\alpha}\\&=\sum_{k=0}^{\infty}\frac{1}{\Gamma(k\alpha+1)}\left(\sum_{m=0}^{k}\binom{k}{m}a_{k-m}b_m\right)x^{k\alpha}.\end{aligned} \tag{3}$$

Equivalently,

$$f_\alpha(x^\alpha)\otimes g_\alpha(x^\alpha)$$

$$= \sum_{k=0}^{\infty} \frac{a_k}{k!} \left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right)^{\otimes k} \otimes \sum_{k=0}^{\infty} \frac{b_k}{k!} \left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right)^{\otimes k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} \left(\sum_{m=0}^{k} \binom{k}{m} a_{k-m} b_m\right) \left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right)^{\otimes k}. \quad (4)$$

Definition 2.3 ([14])**:** If $0 < \alpha \leq 1$, and $f_\alpha(x^\alpha)$ is a α-fractional analytic functions. Then $\left(f_\alpha(x^\alpha)\right)^{\otimes n} = f_\alpha(x^\alpha) \otimes \cdots \otimes f_\alpha(x^\alpha)$ is called the nth power of $f_\alpha(x^\alpha)$.

III. MAIN RESULTS

In this section, we solve a problem involving fractional algebraic equations by using some methods.

Problem 3.1: Let $0 < \alpha \leq 1$, and x, y be real numbers, $x^\alpha \neq y^\alpha$. If the following α-fractional algebraic equations hold:

$$\left[\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes 2} = \frac{1}{\Gamma(\alpha+1)} x^{\alpha} + 1, \quad (5)$$

and

$$\left[\frac{1}{\Gamma(\alpha+1)} y^{\alpha}\right]^{\otimes 2} = \frac{1}{\Gamma(\alpha+1)} y^{\alpha} + 1. \quad (6)$$

Find

$$\left[\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes n} + \left[\frac{1}{\Gamma(\alpha+1)} y^{\alpha}\right]^{\otimes n}, \quad (7)$$

for $n = 1, 2, 3, \cdots, 6$.

Solution Since $\frac{1}{\Gamma(\alpha+1)} x^{\alpha}$ and $\frac{1}{\Gamma(\alpha+1)} y^{\alpha}$ satisfy the α-fractional algebraic equation

$$\left[\frac{1}{\Gamma(\alpha+1)} t^{\alpha}\right]^{\otimes 2} = \frac{1}{\Gamma(\alpha+1)} t^{\alpha} + 1. \quad (8)$$

That is, x^α and y^α are two distinct solutions of the following 2nd order algebraic equation

$$\frac{2}{\Gamma(2\alpha+1)} t^{2\alpha} - \frac{1}{\Gamma(\alpha+1)} t^{\alpha} - 1 = 0. \quad (9)$$

Therefore,

$$x^{\alpha} + y^{\alpha} = -\frac{-\frac{1}{\Gamma(\alpha+1)}}{\frac{2}{\Gamma(2\alpha+1)}} = \frac{\Gamma(2\alpha+1)}{2 \cdot \Gamma(\alpha+1)}, \quad (10)$$

Hence,

$$\frac{1}{\Gamma(\alpha+1)} x^{\alpha} + \frac{1}{\Gamma(\alpha+1)} y^{\alpha} = \frac{\Gamma(2\alpha+1)}{2 \cdot [\Gamma(\alpha+1)]^2}. \quad (11)$$

Furthermore,

$$\left[\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes 2} + \left[\frac{1}{\Gamma(\alpha+1)} y^{\alpha}\right]^{\otimes 2}$$

$$= \frac{1}{\Gamma(\alpha+1)} x^{\alpha} + 1 + \frac{1}{\Gamma(\alpha+1)} y^{\alpha} + 1$$

$$= \frac{\Gamma(2\alpha+1)}{2 \cdot [\Gamma(\alpha+1)]^2} + 2. \quad (12)$$

In addition,

$$\left[\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes 3} + \left[\frac{1}{\Gamma(\alpha+1)} y^{\alpha}\right]^{\otimes 3}$$

$$= \frac{1}{\Gamma(\alpha+1)} x^{\alpha} \otimes \left[\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes 2} + \frac{1}{\Gamma(\alpha+1)} y^{\alpha} \otimes \left[\frac{1}{\Gamma(\alpha+1)} y^{\alpha}\right]^{\otimes 2}$$

$$= \frac{1}{\Gamma(\alpha+1)} x^{\alpha} \otimes \left[\frac{1}{\Gamma(\alpha+1)} x^{\alpha} + 1\right] + \frac{1}{\Gamma(\alpha+1)} y^{\alpha} \otimes \left[\frac{1}{\Gamma(\alpha+1)} y^{\alpha} + 1\right]$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}+\frac{1}{\Gamma(\alpha+1)}y^{\alpha}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}+\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+\frac{1}{\Gamma(\alpha+1)}y^{\alpha}$$

$$=\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}+2+\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}$$

$$=\frac{\Gamma(2\alpha+1)}{[\Gamma(\alpha+1)]^2}+2\,. \tag{13}$$

Moreover,

$$\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 4}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 4}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}\otimes\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}\otimes\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+1\right]\otimes\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+1\right]+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}+1\right]\otimes\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}+1\right]$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}+2\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]+2$$

$$=\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}+2+2\cdot\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}+2$$

$$=\frac{3\cdot\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}+4\,. \tag{14}$$

And

$$\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 5}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 5}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}\otimes\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}\otimes\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}\otimes\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}\otimes\frac{1}{\Gamma(\alpha+1)}y^{\alpha}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+1\right]\otimes\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+1\right]\otimes\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}+1\right]\otimes\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}+1\right]\otimes\frac{1}{\Gamma(\alpha+1)}y^{\alpha}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 3}+2\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}\right]+\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]$$

$$=\frac{\Gamma(2\alpha+1)}{[\Gamma(\alpha+1)]^2}+2+2\left[\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}+2\right]+\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}$$

$$=\frac{5\cdot\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}+6\,. \tag{15}$$

Finally,

$$\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 6}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 6}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+1\right]^{\otimes 3}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}+1\right]^{\otimes 3}$$

$$=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 3}+3\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+\left[\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]^{\otimes 2}\right]+3\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right]+2$$

$$=\frac{\Gamma(2\alpha+1)}{[\Gamma(\alpha+1)]^2}+2+3\left[\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}+2\right]+3\left[\frac{\Gamma(2\alpha+1)}{2\cdot[\Gamma(\alpha+1)]^2}\right]+2$$

$$=\frac{4\cdot\Gamma(2\alpha+1)}{[\Gamma(\alpha+1)]^2}+10\,. \tag{16}$$

IV. CONCLUSION

In this paper, based on a new multiplication of fractional analytic functions, we solve a problem involving fractional algebraic equations. In fact, the fractional algebraic equation is a generalization of ordinary algebraic equation. We can solve this problem by using some methods. In the future, we will continue to use the new multiplication of fractional analytic functions to solve the problems in applied mathematics and fractional differential equations.

REFERENCES

[1] R. L. Magin, Fractional calculus in bioengineering, 13th International Carpathian Control Conference, 2012.

[2] Hasan, A. Fallahgoul, Sergio M. Focardi and Frank J. Fabozzi, Fractional calculus and fractional processes with applications to financial economics, Academic Press, 2017.

[3] Mohd. Farman Ali, Manoj Sharma, Renu Jain, An application of fractional calculus in electrical engineering, Advanced Engineering Technology and Application, vol. 5, no. 2, pp. 41-45, 2016.

[4] R. Hilfer (ed.), Applications of Fractional Calculus in Physics, WSPC, Singapore, 2000.

[5] J. T. Machado, Fractional Calculus: Application in Modeling and Control, Springer New York, 2013.

[6] V. E. Tarasov, Mathematical economics: application of fractional calculus, Mathematics, vol. 8, no. 5, 660, 2020.

[7] C. -H. Yu, A study on fractional RLC circuit, International Research Journal of Engineering and Technology, vol. 7, no. 8, pp. 3422-3425, 2020.

[8] C. -H. Yu, A new insight into fractional logistic equation, International Journal of Engineering Research and Reviews, vol. 9, no. 2, pp.13-17, 2021.

[9] J. F. Douglas, Some applications of fractional calculus to polymer science, Advances in chemical physics, vol 102, John Wiley & Sons, Inc., 2007.

[10] N. Sebaa, Z. E. A. Fellah, W. Lauriks, C. Depollier, Application of fractional calculus to ultrasonic wave propagation in human cancellous bone, Signal Processing archive vol. 86, no. 10, pp. 2668-2677, 2006.

[11] Z. E. A. Fellah, C. Depollier, Application of fractional calculus to the sound waves propagation in rigid porous materials: validation via ultrasonic measurement, Acta Acustica, vol. 88, pp. 34-39, 2002.

[12] C. -H. Yu, Study of fractional analytic functions and local fractional calculus, International Journal of Scientific Research in Science, Engineering and Technology, vol. 8, no. 5, pp. 39-46, 2021.

[13] C. -H. Yu, Study of a problem involving fractional exponential equation, International Journal of Novel Research in Electrical and Mechanical Engineering, vol. 10, no. 1, pp. 63-67, 2023.

[14] C. -H. Yu, Study of a fractional function equation, International Journal of Novel Research in Physics Chemistry and Mathematics, vol. 10, no. 1, pp. 6-9, 2023.